On the generation and structure of the quadrupole magnetic field in the reconnection process: Comparative simulation study

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[1] We investigate magnetic reconnection dynamics using three simulation codes to isolate ion kinetic effects from Hall effects: a full hybrid code, a Hall MHD code, and a new Hall-less hybrid code. The structure of the quadrupole magnetic field (also known as the core field or out-of-plane field) is used as the physical quantity for this comparison because it is a good proxy of fast magnetic reconnection. Examination of the core field time evolution equations for the Hall-less hybrid and Hall MHD models reveal inherent differences between these models. In particular, it demonstrates that core fields can be generated in the kinetic regime even in the absence of the Hall term. We attribute this result to ion kinetic effects, e.g., ion diamagnetic drifts driven by an anisotropic ion stress tensor. This result is borne out by our simulations. Additionally, kinetic ion dynamics leads to core fields with widths that can extend to several ion inertial lengths in the vicinity of *X*-line and tens of ion inertial lengths further away. In contrast, in Hall MHD, the typical case is about an ion inertial length in the vicinity of the *X*-line and a few ion inertial lengths further away. These results pose challenges for improving fluid models of reconnection. *INDEX TERMS:* 2744 Magnetospheric Physics: Magnetotail; 2724 Magnetospheric Physics: Magnetopause, cusp, and boundary layers; 2753 Magnetospheric Physics: Numerical modeling; 7835 Space Plasma Physics: Magnetic reconnection. **Citation:** Karimabadi, H., J. D. Huba, D. Krauss-Varban, and N. Omidi (2004), On the generation and structure of the quadrupole magnetic field in the reconnection process: Comparative simulation study, *Geophys. Res. Lett.*, *31*, L07806, doi:10.1029/2004GL019553.

1. Introduction

[2] Given the dominant role of magnetic reconnection as a transport mechanism in the Earth's magnetosphere, there is a real need to develop accurate models of reconnection that can be incorporated into global MHD simulations of the magnetosphere. To this end, Geospace Environmental Modeling (GEM) challenge 2000 [*Birn et al.*, 2001] involved a concerted effort to compare the properties of reconnection, with a strong emphasis on the reconnection rates, among various models. The key findings of this project were that the Hall term is the minimum physics required to obtain fast reconnection [*Shay et al.*, 2001] and that the reconnection rate is insensitive to the details of the mechanism which breaks the frozen-on condition. Thus, hybrid (fluid electron, kinetic ion), Hall MHD, and full particle codes yielded similar reconnection rates whereas resistive MHD resulted in substantially smaller reconnection rates. There were, however, differences observed in the structure of the reconnection region between Hall MHD and kinetic models [*Hesse et al.*, 2001]. For instance, the width of the ion jet was larger by a factor of 2 in hybrid compared to Hall MHD simulations. So it was concluded that Hall MHD is inadequate to represent the kinetic model on small scales, but it was postulated [*Hesse et al.*, 2001; *Yin and Winske*, 2003] that on larger scales, the difference between Hall MHD and kinetic models may becomes less important.

[3] In this Letter we study the details of the quadrupole magnetic field using a full hybrid code, a Hall MHD code, and a new model: a Hall-less hybrid code. The purpose is to investigate the differences in the kinetic and Hall MHD reconnection dynamics. The prediction of the quadrupole field [*Sonnerup*, 1979; *Terasawa*, 1983], also known as the core field or out-of-plane field, has been one of the great successes of the Hall MHD model and it has generally been assumed that Hall MHD provides a good description of its structure. In fact, experimentalists often use the observation of the core field in the ion diffusion region as an indication of the correctness of Hall MHD description [*Mozer et al.*, 2002]. However, aside from the polarity of this field [*Hesse and Winske*, 1998; *Karimabadi et al.*, 1999; *Pritchett*, 2001], no detailed comparison of the size and structure of the field have been made among different models. We find that the core field is in general much wider in the kinetic regime than that in the Hall MHD. We also show for the first time that a core field can be generated even in the absence of the Hall term and in some cases with the same strength as when there is a Hall term. Implications of these results for understanding the details of reconnection and a summary are presented.

2. Theory

[4] We consider the following plasma and field configuration. The magnetic field at $t = 0$ is in the xy-plane $\mathbf{B} = B_x\hat{\mathbf{e}}_x + B_y\hat{\mathbf{e}}_y$ and the plasma is inhomogeneous in the xy-plane and is stationary. We take Ohm's law to be $\mathbf{E} = -\mathbf{V}_e \times \mathbf{B}/c$ where we have neglected electron inertia and pressure as well as resistivity. Making use of Faraday's law $\partial\mathbf{B}/\partial t = -c\nabla \times \mathbf{E}$ we find that the generation of the out-of-plane field (or core field) B_z is given by

$$\partial B_z/\partial t + [(\nabla \cdot \mathbf{V}_e) + (\mathbf{V}_e \cdot \nabla)]B_z = (\mathbf{B} \cdot \nabla)V_{ez} \quad (1)$$

Thus, the generation of the out-of-plane magnetic field is controlled by electron dynamics. A differential flow of the

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electrons in the *xy*-plane 'drags' the magnetic field non-uniformly in the ±*z*-directions (see Figure 2 of *Huba and Rudakov* [2002]). Since B_z is initially zero, the two terms that are proportional to B_z in (1) do not play a significant role in the initial generation of B_z. In Hall MHD, as well as in hybrid simulation codes, we write $\mathbf{V}_e = \mathbf{V}_i - \mathbf{J}/ne$ so that (1) can be written as

$$\frac{\partial B_z}{\partial t} = (\mathbf{B} \cdot \nabla)\left(V_{iz} - \underbrace{J_z/ne}_{\text{Hall term}}\right) \quad (2)$$

where the Hall term is explicitly identified and we have ignored the terms proportional to B_z. In reconnection dynamics the Hall term is responsible for the out-of-plane field generation; in ideal MHD B_z can never be generated if $V_{iz} = 0$ at $t = 0$. However, this is not necessarily the case when full ion kinetic dynamics is considered. To show this we have developed a Hall-less hybrid simulation code. The ion equation is given by $m_i d\mathbf{v}_i/dt = e(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}/c)$ and the Ohm's law used in this equation is $\mathbf{E} = -\mathbf{V}_i \times \mathbf{B}/c + \mathbf{J} \times \mathbf{B}/nec$. However, in Faraday's law we use $\mathbf{E}' = -\mathbf{V}_i \times \mathbf{B}/c$, i.e., we neglect the Hall term in the evolution of the magnetic field. [In the Hall-less hybrid code, we do keep the pressure and resistivity terms in the Ohm's law but since neither of these two effects play a significant role in the direct generation of B_z, we will ignore them in our discussions here.] Then the out-of-plane field generation is simply given by

$$\frac{\partial B_z}{\partial t} = (\mathbf{B} \cdot \nabla)V_{iz} \quad (3)$$

so that differential motion of the ion flow generates B_z. Such a flow can develop in hybrid simulations through ion kinetic effects. From a fluid point of view, this can occur through the development of an anisotropic ion stress tensor that produces ion diamagnetic drifts [*Braginskii*, 1965]. Thus, there appear to be two ways to generate B_z: an inhomogeneous ion flow and an inhomogeneous current (the usual Hall term mechanism). Given that the evolution equation for V_{iz} is different in Hall MHD and the hybrid code, we would expect differences in the structure of the resulting B_z. Another prediction resulting from (1) is in regards to the effect of T_e/T_i on B_z. In the Harris equilibrium $V_{iz}/V_{ez} = T_i/T_e$. Since B_z is driven by V_{ez}, it follows that the magnitude of B_z correlates with T_e/T_i. This is consistent with findings of *Karimabadi et al.* [1999] that colder ions lead to larger core fields.

3. Simulation Models

[5] We use three types of two-dimensional simulation codes: (a) Hall MHD [*Huba*, 2003], (b) full hybrid, and (c) a Hall-less hybrid. The Hall-less hybrid code is a new variation of the hybrid code that eliminates the Hall term in the field update equations (H. Karimabadi et al., On magnetic reconnection regimes and associated three-dimensional asymmetrics: Hybrid, Hall-less hybrid, and Hall-MHD simulations, submitted to *Journal of Geophysical Research*, 2004), as discussed in the previous section. The coordinate system used is such that the main component of the reversed magnetic field is in the *y*-direction with spatial variations in the *x*-direction, and the current supporting this field is in the *z*-direction. Time and space are normalized to the ion cyclotron frequency $\Omega_i = eB_0/m_i c$ and the ion inertial length c/ω_{pi} where $\omega_{pi} = (4\pi n_0 e^2/m_i)^{1/2}$, B_0 is the magnetic field far from the neutral line and n_0 is the density at the neutral line at $t = 0$. The simulations use a Harris-like equilibrium. The magnetic field is $\mathbf{B} = B_0 \tanh(x/L_x)\,\hat{e}_y$. The equilibrium satisfies $B_0^2(x)/8\pi + n(x)T =$ constant. The density has a maximum value n_0 in the center of the current sheet at $x = 0$ and a minimum value n_1 away from the current sheet at $|x| > L_x$.

[6] In the hybrid simulations this is accomplished by considering two ion species: equilibrium ions *i* and background ions *b*. All boundaries have freely floating field conditions that allow the magnetic field to slip as necessary. Likewise, for the ions the boundaries are of free inflow-outflow type. The typical grid size in the kinetic runs is $\Delta = 0.1$ in each direction with 200–1600 particles per cell. The typical simulation box is 50×200. The time step is $\Delta t = 0.001$–0.004. We take $\rho_i/L_x = 0.8$, $T_e/T_i = 0.2$, and $T_b = T_i$ where T_b and T_i are the temperature of the background and equilibrium ions, respectively, and ρ_i is the ion gyroradius. In the hybrid simulations we impose a localized resistivity (normalized to $4\pi/\omega_{pi}$) of 4×10^{-4}. We consider a box-shaped rectangular region of 1×2 at the center of the box and a cosh profile for the resistivity that extends to 2×2. The Hall MHD fluid code uses a 80×160 mesh that is nonuniform; there are roughly 30 grid points within the current layer. The extent of the system is 98×188. The simulation is initialized with a magnetic perturbation similar to that used in the GEM reconnection study and is run to time $t = 114$.

4. Simulation Results

[7] In order to gain a better understanding of the core field generation in the kinetic limit, we first show the core field structure as obtained from a high resolution hybrid simulation for a case with the box shaped resistivity. Figure 1 shows the intensity plot of the core field B_z at an early time in the run. The quadrupole structure in B_z is seen to be comprised of two distinct spatial structures. The outer and inner structures are about 0.25 and 0.6 c/ω_{pi} wide, respec-

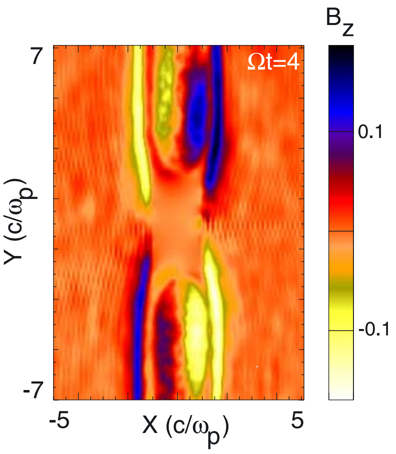

Figure 1. Intensity plot of B_z at an early time from a hybrid simulation. Note that two distinct structures are clearly visible.

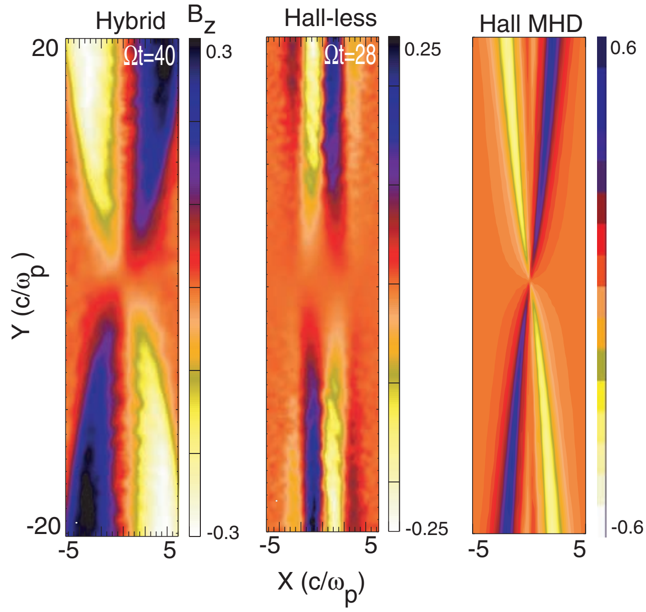

Figure 2. Comparison of the B_z structure from a hybrid, Hall-less hybrid and Hall MHD simulation. In each case the system has reached a steady state.

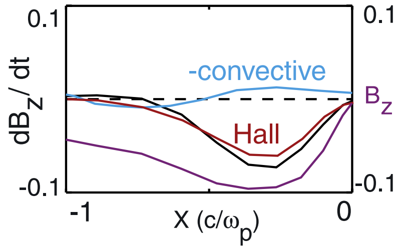

Figure 4. 1D cuts of B_z, $\partial B_z/\partial t$, and the convective and the Hall contributions to $\partial B_z/\partial t$ at an early time from the Hall MHD simulation shown in Figure 2.

tively. These structures grow in time and eventually merge, spreading over 4 c/ω_{pi} in width. This figure clearly demonstrates that there are at least two different processes involved in the B_z generation.

[8] To investigate this further and to gain further insight into differences between the Hall MHD and kinetic structures of B_z, we show intensity plots of B_z in the *xy* plane after the system has reached a steady state for the Hall MHD, full hybrid, and Hall-less hybrid cases in Figure 2. Here we use the cosh profile for resistivity in the hybrid and Hall-less hybrid simulations. Two points are immediately clear. First, even in the absence of the Hall term, a quadrupole B_z is still generated. This field has almost the same magnitude as in the hybrid case but is not as wide. This is partly due to the fact that the reconnection cone angle becomes smaller in the absence of the Hall term. And second, B_z is largest in magnitude but thinnest in width for Hall MHD. The change in the half-width of B_z as a function of distance *y* away from the *X*-line is shown in Figure 3 from the same three runs. In general the half-width increases as one moves farther away from the *X*-line and the half-width of B_z is seen to be about a factor of 2 smaller in Hall MHD as compared to hybrid. In the Hall-less hybrid case, however, the width of B_z is nearly constant as a function of *y*. This is due to the fact that the reconnection cone becomes very small in the absence of the Hall term in this case.

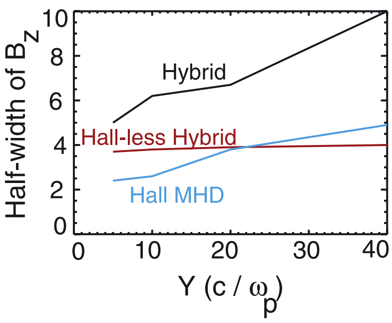

Figure 3. Variation of the half-width of B_z with the distance from the *X*-line from the simulation results shown in Figure 2.

[9] Next, we examine the time evolution of B_z. Figure 4 shows a 1D cut of B_z from the Hall MHD simulation at early time. Also shown are $\partial B_z/\partial t$ and its two components in (2): the convective term and the Hall term. We have reversed the sign of the convective term to better compare its strength with the Hall term. At early times, the Hall term is seen to dominate $\partial B_z/\partial t$ and is the main driving force in the generation of B_z. However, at late times the convective part in $\partial B_z/\partial t$ can become comparable to the Hall term. Although the ion velocity was zero initially in the Hall MHD, there arises a differential ion flow (unlike in MHD); this contributes to the evolution of B_z (and can cancel the Hall-generated term, in particular, in the inflow regions). In the kinetic case, the early generation of B_z (Figure 5) shows two distinct structures similar to that in Figure 1 but now the inner structure has an opposite polarity to the usual quadrupole structure. This is because there are two competing processes that come in through the B_z term in (2). One is due to the initial differential ion flow V_{iz} associated with the

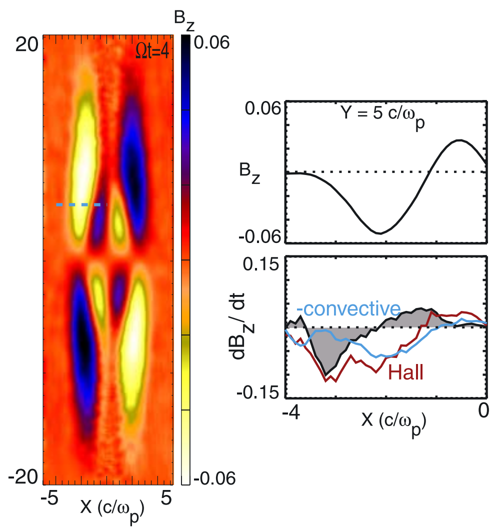

Figure 5. (a) Intensity plot of B_z and (b) 1D cuts of B_z, $\partial B_z/\partial t$, and the convective and Hall contributions to $\partial B_z/\partial t$ at an early time from the hybrid simulation shown in Figure 2.

equilibrium (Harris equilibrium with a background), the other is the modification to the equilibrium V_{iz} due to FLR and other effects. The former leads to a B_z with an opposite polarity to the Hall B_z, while the latter generates the same polarity as the Hall term. In the present case, the onset of reconnection generates a finite B_x which together with the equilibrium V_{iz} lead to the formation of a B_z structure that has an opposite polarity to the usual quadrupole structure. The formation of this structure depends on the value and extent of the resistivity. If the initial current is mostly in the electrons, $T_e \gg T_i$, then V_{iz} adjusts locally and can take on an opposite sign to the original V_{iz}. In the opposite limit where most of the current is in the ions, the terms proportional to B_z on the left hand side of (1) eventually dominate and control the evolution of B_z. Another difference with the Hall MHD is that even at early times the convective term in $\partial B_z/\partial t$ can be sufficiently large to affect the structure of B_z. This is seen in Figure 5 where the Hall term is seen to dominate in the outside region whereas the convective term becomes larger in the inner region.

5. Conclusion

[10] We have used a new approach to understand the differences between Hall MHD and kinetic models of reconnection. We used the structure of the core field, which is a good proxy of the fast reconnection process, as the physical quantity for our comparison. Through the use of a new model, the Hall-less hybrid code, we were then able to isolate the kinetic effects from Hall effects. First, the core field in the kinetic limit is, in general, wider than that in Hall MHD. The typical width in the Hall MHD is on the order of an ion inertial length whereas in the kinetic regime it can extend to tens of ion inertial lengths. The differences between the kinetic and Hall MHD remain significant even at distances far from the *X*-line. And second, kinetic effects can also create a quadrupole structure and depending on the parameter regime this field can be as large as those in the presence of the Hall term. Third, the observed differences in B_z between kinetic and Hall MHD simulations is closely related to differences in the time evolution equation of V_{iz} in Hall MHD and hybrid regimes. In Hall MHD, the equation for V_{iz} is not complete. For instance it is missing FLR effects which can generate B_z. In the kinetic regime, V_{iz} evolves and remains finite even in the absence of the Hall term. It will be interesting to see the extent to which inclusion of FLR effects (i.e., anisotropic ion stress tensor) in a fluid description will modify these fluid results. A comparative study of the Rayleigh-Taylor instability using hybrid and a Hall/FLR codes demonstrated good correspondence between the kinetic and fluid descriptions [*Huba and Winske*, 1998]. At rotational discontinuities, we found pressure anisotropies as well as off-diagonal terms to be important. However, the double-adiabatic approximation commonly used in conjunction with FLR fluid extensions gave inconsistent results [*Krauss-Varban et al.*, 1995].

[11] Ultimately, the evolution of the magnetic field is controlled by the electron fluid. However, ion dynamics play a major role in the redistribution of the plasma and impact the electron fluid via quasi-neutrality. To study reconnection self-consistently on ion inertial length scale lengths, the Hall term must be included in both fluid and hybrid codes. The purpose of the Hall-less code is to isolate ion kinetic effects to better understand their impact on the system. We plan to use this code to examine the effect of the absence of the Hall term in the reconnection rate as well as studies of collisionless shocks. Finally, given the large differences in the structure of B_z and E_z (not shown) in the Hall MHD and kinetic description, it becomes relevant to test these predictions in satellite observations.

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